TRINITY COLLEGE

Mathematics Specialist Units 1,2
Test 3   2017

Calculator Assumed
Proof, Vector Proof, Circle Geometry

STUDENT'S NAME   _____

DATE: Friday 19 May   **TIME:** 60 minutes   **MARKS**: 52

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this
                         assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (6 marks)

Consider the true statement:
"If quadrilateral ABCD is a rhombus, then it is a parallelogram"

(a)   Write down the converse of this statement and state whether it is true or false, and if it
         is false, provide a counter-example.   [2]

(b)   Write down the contrapositive of this statement and state whether it is true or false, and
         if it is false, provide a counter-example.   [2]

(c)   Write down the inverse of this statement and state whether it is true or false, and if it is
         false, provide a counter-example.   [2]

2.   (8 marks)

In this diagram, AOB is the diameter of a circle, AC is a chord of the circle and CD is perpendicular to the tangent AD.
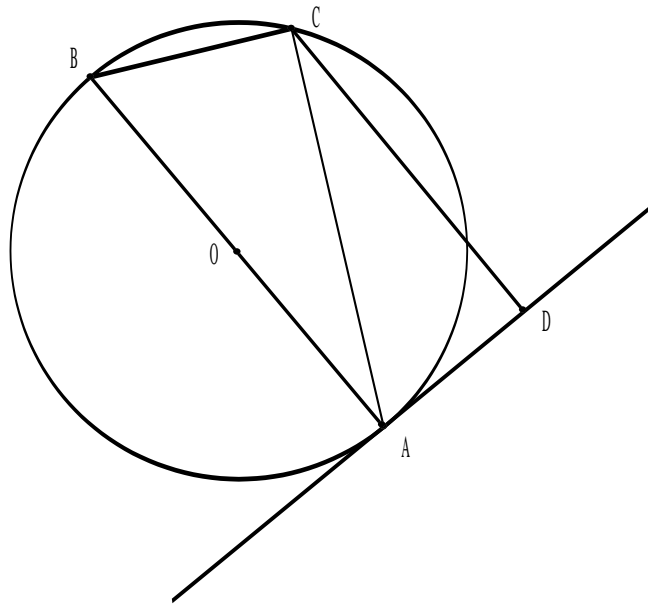
(a)   Prove ΔABC is similar to ΔCAD [3]

(b)   Hence show $(AC)^2 = AB.CD$ [2]

(c)   Determine the radius of the circle when $AC = 15\ cm$ and $AD = 12\ cm$.   [3]

3. (6 marks)

    $OABC$ is a parallelogram with $\overrightarrow{OA} = a$ and $\overrightarrow{OC} = c$. M is the midpoint of the diagonal OB.

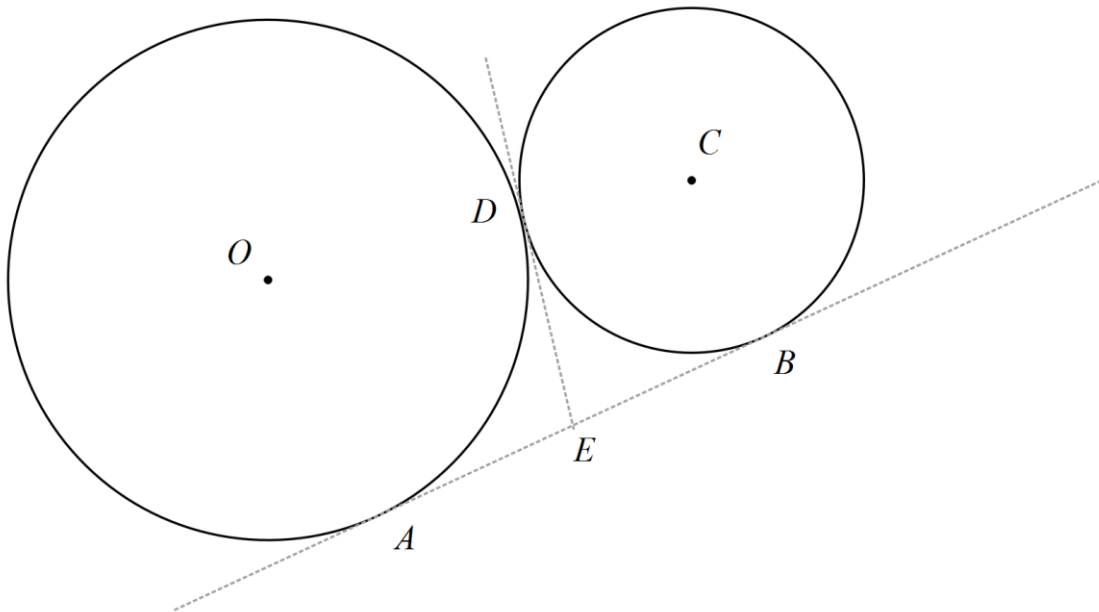   (a)   Determine $\overrightarrow{CM}$ in terms of a and c [2]

    (b)   Determine $\overrightarrow{CA}$ in terms of a and c [1]

    (c)   Hence show that M lies on $\overrightarrow{CA}$ and is the midpoint of $\overrightarrow{CA}$.   [3]

4.   (10 marks)

The circle with centre *O* and the circle with centre *C* meet externally at *D* so that *DE* is a common tangent and *AB* is a tangent to both circles.

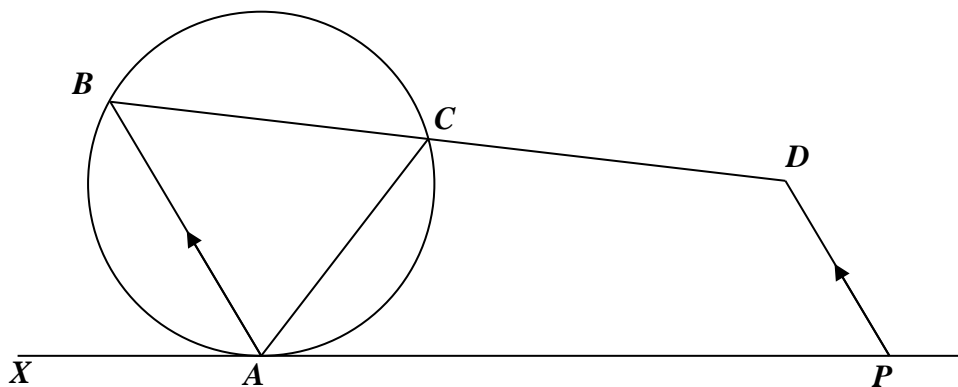(a)   Prove *O*, *D* and *C* are collinear.   [3]

(b)   Prove the common tangent at *D* bisects *AB*.   [3]
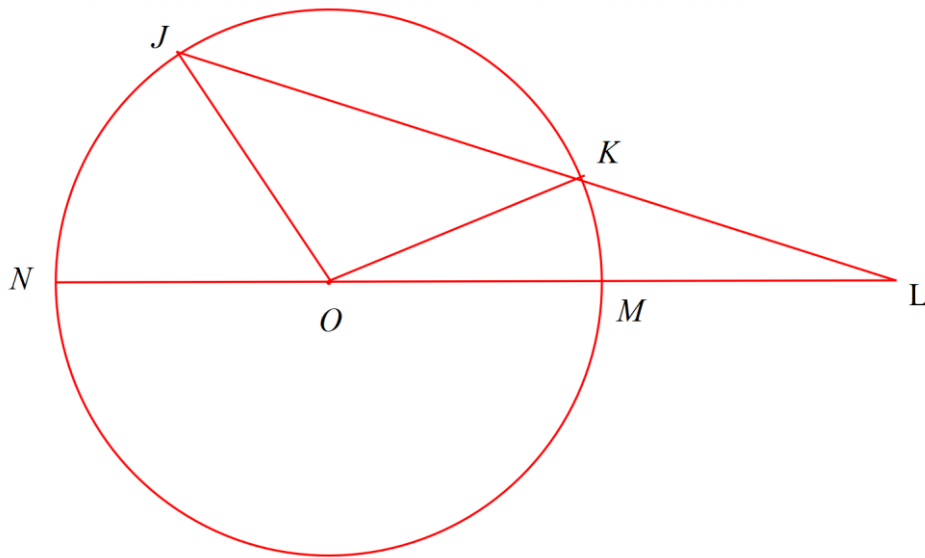
(c)   Prove $\angle ADB = 90°$.   [4]

5.   (7 marks)

In the diagram, *AP* is a tangent to the circle, *D* is the point on *BC* produced such that *AB* is parallel to *PD*.

Prove that *ACDP* is a cyclic quadrilateral.

6.   (7 marks)

In the diagram above, O is the centre of the circle. *LMON* and *JKL* are straight lines.
Let $\angle JON = \beta$ and $\angle KLM = \theta$.
The length *KL* is equal to the radius of the circle.

Prove that $\beta = 3\theta$.

7. (8 marks)

Parallelogram OABD has C on $\overrightarrow{DB}$ such that $\overrightarrow{DC} = \dfrac{3}{5}\overrightarrow{DB}$ and E on $\overrightarrow{OD}$ such that $\overrightarrow{OE} = \dfrac{2}{3}\overrightarrow{OD}$.

Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OD} = \underset{\sim}{d}$, $\overrightarrow{OP} = h\overrightarrow{OC}$ and $\overrightarrow{AP} = k\overrightarrow{AE}$ where P is the point of intersection of $\overrightarrow{AE}$ and $\overrightarrow{OC}$.

Determine the values of h and k.

A
B
O
D